# ECL 4340

## POWER SYSTEMS

## LECTURE 16

SYMMETRICAL FAULTS, FAULT CURRENT

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## ANNOUNCEMENTS

- Be reading Chapter 7.
- HW 9 is uploaded, due November 11, Friday.
- Exam II is on November 8, Tuesday.

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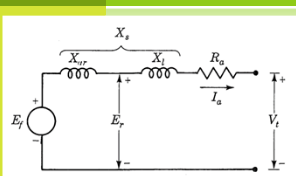
## GENERATOR MODELING




Equivalent Circuit

Terminal Voltage:

$$V_t = E_r - jI_aX_l$$

$$V_t = E_f - I_a(R_a + jX_s)$$

$X_s = X_{ar} + X_l$ : synchronous reactance

$R_a$ : armature resistance

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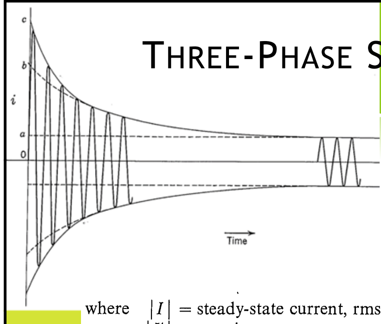
## Three-Phase Short Circuit




$$|I| = \frac{oa}{\sqrt{2}} = \frac{|E_g|}{X_d}$$

$$|I'| = \frac{ob}{\sqrt{2}} = \frac{|E_g|}{X'_d}$$

$$|I''| = \frac{oc}{\sqrt{2}} = \frac{|E_g|}{X''_d}$$

where $|I|$ = steady-state current, rms value
$|I'|$ = transient current, rms value excluding dc component
$|I''|$ = subtransient current, rms value excluding dc component
$X_d$ = direct-axis synchronous reactance
$X'_d$ = direct-axis transient reactance
$X''_d$ = direct-axis subtransient reactance
$|E_g|$ = rms voltage from one terminal to neutral at no load
$oa, ob, oc$ = intercepts shown in Fig. 10.3.

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## Three-Phase Short Circuit

Theorem of Constant Flux Linkages:

Flux linkage cannot change instantaneously.
Initially forced to flow through high reluctance path, i.e., low reactance path.

Reluctance: $\mathcal{R}$

Flux: $\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{NI}{\mathcal{R}} \stackrel{d}{=} LI$

Then moves towards the lower reluctance path, i.e., high reactance path.

$\Rightarrow$ Time-varying inductance $L(t)$
or reactance $X(t) = \omega L(t)$.

Define: $X''_d < X'_d < X_d$

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## Three-Phase Short Circuit

Instantaneous fault current:

$$i_{ac}(t) = \sqrt{2}E_g\left[\left(\frac{1}{X''_d} - \frac{1}{X'_d}\right)e^{-t/T''_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d}\right)e^{-t/T'_d} + \frac{1}{X_d}\right]\sin\left(\omega t + \alpha - \frac{\pi}{2}\right)$$

at $t=0$: $I_{ac}(0) = \frac{E_g}{X''_d} = I''$, subtransient current, $\theta = \frac{\pi}{2}$ for $R=0$

$T''_d$, d-axis subtransient current time constant

for large $t$, $I'_{ac} = \frac{E_g}{X'_d}$, transient current,

for much larger $t$, $I_{ac}(\infty) = \frac{E_g}{X_d} = I$, steady-state current

Max dc offset: @$\alpha=0$, $i_{dc\,max}(t) = \frac{\sqrt{2}E_g}{X''_d} e^{-t/T_A} = \sqrt{2} I'' e^{-t/T_A}$
= peak of ac @ $t=0$.

$T_A$: armature time constant

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## Generator Short Circuit Example

- A 500 MVA, 20 kV, 3ϕ is operated with an internal voltage of 1.05 pu. Assume a solid 3ϕ fault occurs on the generator's terminal and that the circuit breaker operates after three cycles. Determine the fault current. Assume

$$X_d'' = 0.15, \quad X_d' = 0.24, \quad X_d = 1.1 \text{ (all per unit)}$$
$$T_d'' = 0.035 \text{ seconds}, \; T_d' = 2.0 \text{ seconds}$$
$$T_A = 0.2 \text{ seconds}$$

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## Generator S.C. Example, cont'd

Substituting in the values

$$I_{ac}(t) = 1.05\left[\frac{1}{1.1} + \left(\frac{1}{0.24} - \frac{1}{1.1}\right)e^{-t/2.0} + \left(\frac{1}{0.15} - \frac{1}{0.24}\right)e^{-t/0.035}\right]$$

$$I_{ac}(0) = 1.05/0.15 = 7 \text{ p.u.}$$

$$I_{base} = \frac{500\times10^6}{\sqrt{3}\,20\times10^3} = 14{,}433 \text{ A} \quad I_{ac}(0) = 101{,}000 \text{ A}$$

$$I_{DC}(0) = 101 \text{ kA} \times \sqrt{2}\, e^{t/0.2} = 143 \text{ k A} \quad I_{RMS}(0) = 175 \text{ kA}$$

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## Generator S.C. Example, cont'd

Evaluating at t = 0.05 seconds for breaker opening

$$I_{ac}(0.05) = 1.05\left[\frac{1}{1.1} + \left(\frac{1}{0.24} - \frac{1}{1.1}\right)e^{-0.05/2.0} + \left(\frac{1}{0.15} - \frac{1}{0.24}\right)e^{-0.05/0.035}\right]$$

$$I_{ac}(0.05) = 70.8 \text{ kA}$$

$$I_{DC}(0.05) = 143 \times e^{-0.05/0.2} \text{ kA} = 111 \text{ k A}$$

$$I_{RMS}(0.05) = \sqrt{70.8^2 + 111^2} = 132 \text{ kA}$$

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## Network Fault Analysis Simplifications

- To simplify analysis of fault currents in networks we'll make several simplifications:
  1. Transmission lines are represented by their series reactance
  2. Transformers are represented by their leakage reactances
  3. Synchronous machines are modeled as a constant voltage behind direct-axis subtransient reactance
  4. Induction motors are ignored or treated as synchronous machines
  5. Other (nonspinning) loads are ignored

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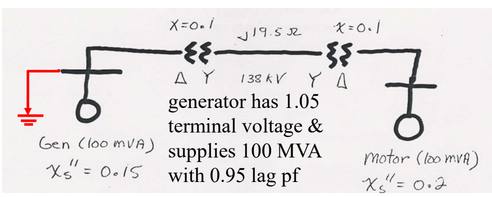
## Network Fault Example

For the following network assume a fault on the terminal of the generator; all data is per unit except for the transmission line reactance




Convert to per unit: $X_{line} = \frac{19.5}{138^2/100} = 0.1$ per unit

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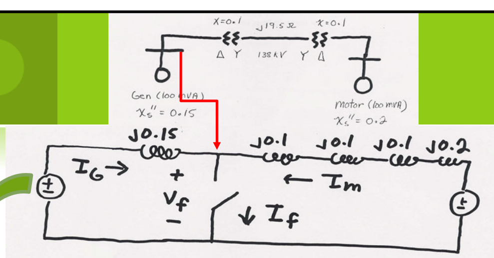


To determine the fault current, we need to first estimate the internal voltages for the Generator and Motor.

For the generator $V_T = 1.05,\ \ S_G = 1.0\angle 18.2°$

$$I_{Gen} = \left(\frac{1.0\angle 18.2}{1.05}\right)^* = 0.952\angle -18.2° = 0.9044\ -\ j0.2973$$

$$E''_g = 1.05\angle 0° + (0.9044\ -\ j0.2973) \times j0.15 = 1.103\angle 7.1°$$

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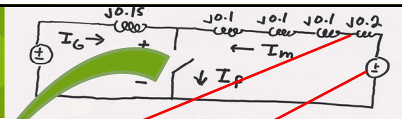
## Cont'd




The motor's terminal voltage is then

$$V_m = 1.05\angle 0 - (0.9044 - j0.2973) \times j0.3 = 1.00\angle -15.8°$$

The motor's internal voltage is

$$E''_m = 1.00\angle -15.8° - (0.9044 - j0.2973) \times j0.2$$
$$= 1.008\angle -26.6°$$

We can then solve as a linear circuit:

$$I_f = I_g + I_m = \frac{1.103\angle 7.1°}{j0.15} + \frac{1.008\angle -26.6°}{j0.5}$$
$$= 7.353\angle -82.9° + 2.016\angle -116.6° = j9.09$$

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## Fault Analysis Solution Techniques

- Circuit models used during the fault allow the network to be represented as a linear circuit
- There are two main methods for solving for fault currents:
  1. **Direct method**: Use pre-fault conditions to solve for the internal machine voltages; then apply fault and solve directly
  2. **Superposition**: Fault is represented by two opposing voltage sources; solve system by superposition
     - first voltage just represents the prefault operating point
     - second system only has a single voltage source

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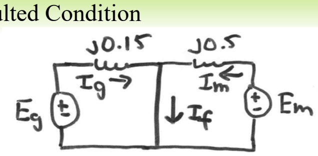
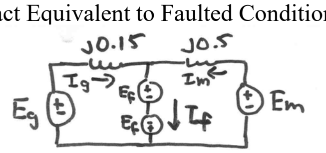
## Superposition Approach

Faulted Condition




Exact Equivalent to Faulted Condition




Fault is represented by two equal and opposite voltage sources, each with a magnitude equal to the pre-fault voltage

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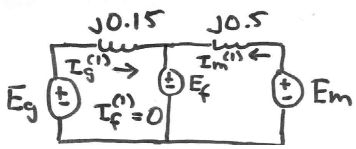
## Superposition Approach, cont'd

Since this is now a linear network, the faulted voltages and currents are just the sum of the pre-fault conditions [the (1) component] and the conditions with just a single voltage source at the fault location [the (2) component]

Pre-fault (1) component equal to the pre-fault power flow solution




Obviously the pre-fault "fault current" is zero!

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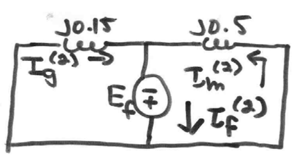
## Superposition Approach, cont'd

Fault (2) component due to a single voltage source at the fault location, with a magnitude equal to the negative of the pre-fault voltage at the fault location.




$$I_g = I_g^{(1)} + I_g^{(2)} \qquad I_m = I_m^{(1)} + I_m^{(2)}$$

$$I_f = I_f^{(1)} + I_f^{(2)} = 0 + I_f^{(2)}$$

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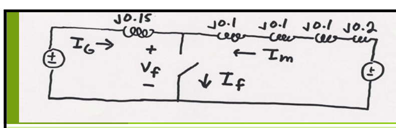
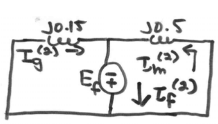



## Superposition

Before the fault we had $E_f = 1.05\angle 0°$,

$I_g^{(1)} = 0.952\angle -18.2°$ and $I_m^{(1)} = -0.952\angle -18.2°$

Solving for the (2) network we get




$$I_g^{(2)} = \frac{E_f}{j0.15} = \frac{1.05\angle 0°}{j0.15} = -j7$$

$$I_m^{(2)} = \frac{E_f}{j0.5} = \frac{1.05\angle 0°}{j0.5} = -j2.1$$

$$I_f^{(2)} = -j7 - j2.1 = -j9.1$$

$$I_g = 0.952\angle -18.2° - j7 = 7.35\angle -82.9°$$

This matches what we calculated earlier

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## Extension to Larger Systems

The superposition approach can be easily extended to larger systems. Using the $\mathbf{Y}_{bus}$ we have

$$\mathbf{Y}_{bus}\mathbf{V} = \mathbf{I}$$

For the second (2) system there is only one voltage source so **I** is all zeros except at the fault location

$$\mathbf{I} = \begin{bmatrix} \vdots \\ 0 \\ -I_f \\ 0 \\ \vdots \end{bmatrix}$$

However to use this approach we need to first determine $I_f$

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## Determination of Fault Current

Define the bus impedance matrix $\mathbf{Z}_{bus}$ as

$$\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1} \qquad \mathbf{V} = \mathbf{Z}_{bus}\mathbf{I}$$

Then

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & \ddots & \vdots \\ Z_{n1} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} \vdots \\ 0 \\ -I_f \\ 0 \\ \vdots \end{bmatrix} = \begin{bmatrix} V_1^{(2)} \\ V_2^{(2)} \\ \vdots \\ V_{n-1}^{(2)} \\ V_n^{(2)} \end{bmatrix}$$

i-th

For a fault at bus i we get $-I_f Z_{ii} = -V_f = -V_i^{(1)}$

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## Determination of Fault Current

Hence

$$I_f = \frac{V_i^{(1)}}{Z_{ii}} \qquad V_j^{(2)} = Z_{ji}(-I_f) = -\frac{Z_{ij}}{Z_{ii}}V_i^{(1)}$$

Where

$Z_{ii}$ = driving point impedance

$Z_{ij} (i \neq j)$ = transfer point imepdance

Voltages during the fault are also found by superposition

$V_i = V_i^{(1)} + V_i^{(2)}$ $\qquad V_i^{(1)}$ are prefault values

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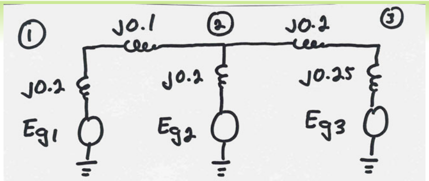
## Three-Gen System Fault Example




For simplicity assume the system is unloaded before the fault with

$$E_{g1} = E_{g2} = E_{g3} = 1.05\angle 0°$$

Hence all the prefault currents are zero.

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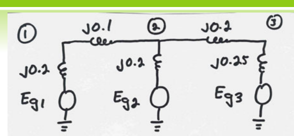
## Three-Gen Example, cont'd

$$\mathbf{Y}_{bus} = j\begin{bmatrix} -15 & 10 & 0 \\ 10 & -20 & 5 \\ 0 & 5 & -9 \end{bmatrix}$$




$$Z_{bus} = j\begin{bmatrix} -15 & 10 & 0 \\ 10 & -20 & 5 \\ 0 & 5 & -9 \end{bmatrix}^{-1}$$

$$= j\begin{bmatrix} 0.1088 & 0.0632 & 0.0351 \\ 0.0632 & 0.0947 & 0.0526 \\ 0.0351 & 0.0526 & 0.1409 \end{bmatrix}$$

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## Three-Gen Example, cont'd

For a fault at bus 1 we get $I_1 = \dfrac{1.05}{j0.1088} = -j9.6 = I_f$

$$\mathbf{V}^{(2)} = j\begin{bmatrix} 0.1088 & 0.0632 & 0.0351 \\ 0.0632 & 0.0947 & 0.0526 \\ 0.0351 & 0.0526 & 0.1409 \end{bmatrix}\begin{bmatrix} j9.6 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1.05\angle 0° \\ -0.60\angle 0° \\ -0.337\angle 0° \end{bmatrix}$$

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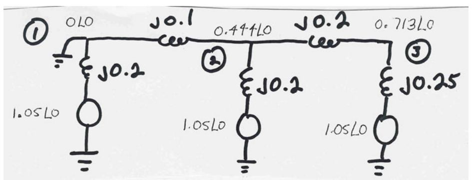
## Three-Gen Example, cont'd

$$\mathbf{V} = \begin{bmatrix} 1.05\angle 0^\circ \\ 1.05\angle 0^\circ \\ 1.05\angle 0^\circ \end{bmatrix} + \begin{bmatrix} -1.05\angle 0^\circ \\ -0.606\angle 0^\circ \\ -0.337\angle 0^\circ \end{bmatrix} = \begin{bmatrix} 0\angle 0^\circ \\ 0.444\angle 0^\circ \\ 0.713\angle 0^\circ \end{bmatrix}$$

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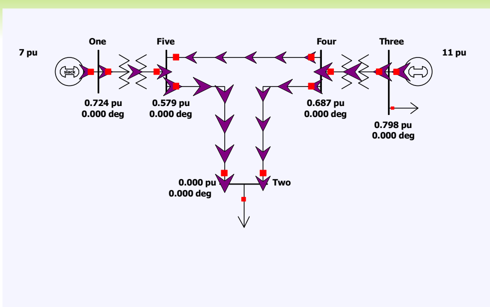
## PowerWorld Example 7.5: Bus-2 Fault

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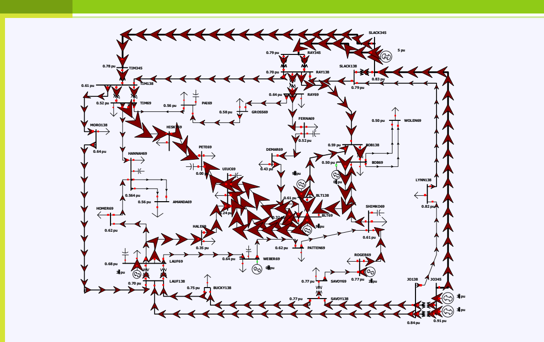
## Problem 7.28



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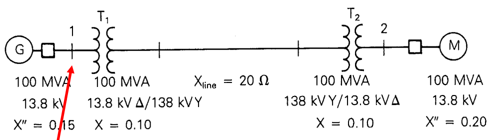
## Example 7.3




The synchronous generator in Figure 7.3 is operating at rated MVA, 0.95 p.f. lagging and at 5% above rated voltage when a bolted three-phase short circuit occurs at bus 1. Calculate the per-unit values of (a) subtransient fault current; (b) subtransient generator and motor currents, neglecting prefault current; and (c) subtransient generator and motor currents including prefault current.

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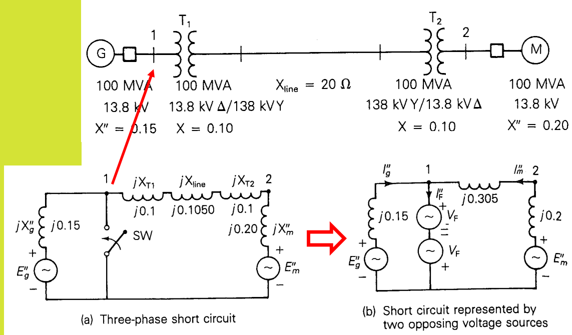
## Example 7.3, continued

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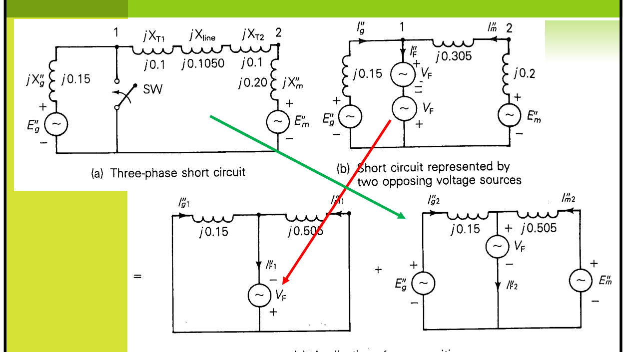
## Example 7.3, continued

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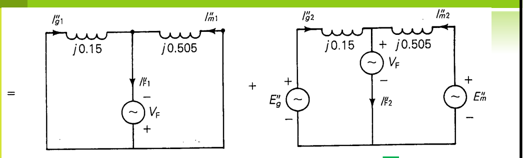
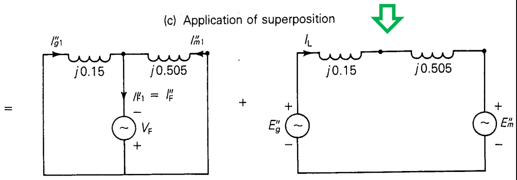
# EXAMPLE 7.3, CONTINUED




(c) Application of superposition




(d) $V_F$ set equal to prefault voltage at fault

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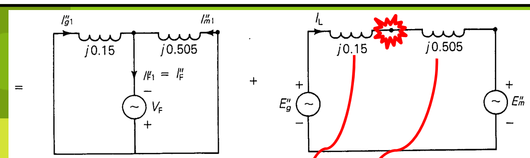


(d) $V_F$ set equal to prefault voltage at fault

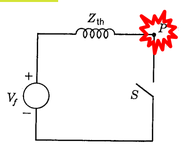


Figure 7.4(d), the Thévenin impedance as viewed from the fault is

$$Z_{Th} = jX_{Th} = j\frac{(0.15)(0.505)}{(0.15+0.505)} = j0.11565 \quad \text{per unit}$$

and the prefault voltage at the generator terminals is

$$V_F = 1.05\underline{/0^\circ} \quad \text{per unit}$$

The subtransient fault current is then

$$I''_F = \frac{V_F}{Z_{Th}} = \frac{1.05\underline{/0^\circ}}{j0.11565} = -j9.079 \quad \text{per unit}$$

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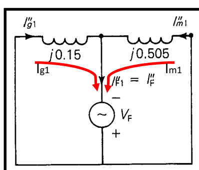
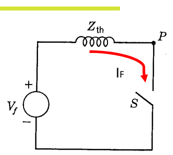



# EXAMPLE 7.3, CONTINUED




Thévenin impedance as viewed from the fault is

$$Z_{Th} = jX_{Th} = j\frac{(0.15)(0.505)}{(0.15+0.505)} = j0.11565 \quad \text{per unit}$$

and the prefault voltage at the generator terminals is

$$V_F = 1.05\underline{/0^\circ} \quad \text{per unit}$$

The subtransient fault current is then

$$I''_F = \frac{V_F}{Z_{Th}} = \frac{1.05\underline{/0^\circ}}{j0.11565} = -j9.079 \quad \text{per unit}$$

Using current division in the first circuit of Figure 7.4(d),

$$I''_{g1} = \left(\frac{0.505}{0.505+0.15}\right) I''_F = (0.7710)(-j9.079) = -j7.000 \quad \text{per unit}$$

$$I''_{m1} = \left(\frac{0.15}{0.505+0.15}\right) I''_F = (0.2290)(-j9.079) = -j2.079 \quad \text{per unit}$$

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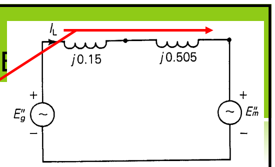
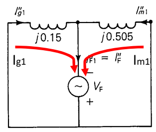
## POWER SYSTEM 3-PHASE




c. The generator base current is

$$I_{\text{base, gen}} = \frac{100}{(\sqrt{3})(13.8)} = 4.1837 \quad \text{kA}$$

and the prefault generator current is




$$I_L = \frac{100}{(\sqrt{3})(1.05 \times 13.8)} \angle -\cos^{-1} 0.95 = 3.9845 \angle -18.19° \quad \text{kA}$$

$$= \frac{3.9845\angle -18.19°}{4.1837} = 0.9524 \angle -18.19°$$

$$= 0.9048 - j0.2974 \quad \text{per unit}$$

The subtransient generator and motor currents, including prefault current, are then

$$I''_g = I''_{g1} + I_L = -j7.000 + 0.9048 - j0.2974$$

$$= 0.9048 - j7.297 = 7.353 \angle -82.9° \quad \text{per unit}$$

$$I''_m = I''_{m1} - I_L = -j2.079 - 0.9048 + j0.2974$$

$$= -0.9048 - j1.782 = 1.999 \angle 243.1° \quad \text{per unit}$$

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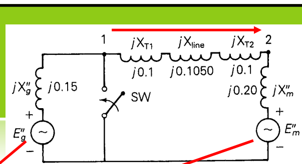
## PROBLEM 7.11




(a) Three-phase short circuit

An alternate method of solving Example 7.3 is to first calculate the internal voltages $E''_g$ and $E''_m$ using the prefault load current $I_L$. Then, instead of using superposition, the fault currents can be resolved directly from the circuit in Figure 7.4(a) (see Problem 7.11). However, in a system with many synchronous machines, the superposition method has the advantage that all machine voltage sources are shorted, and the prefault voltage is the only source required to calculate the fault current. Also, when calculating the contributions to fault current from each branch, prefault currents are usually small, and hence can be neglected. Otherwise, prefault load currents could be obtained from a power-flow program. ■

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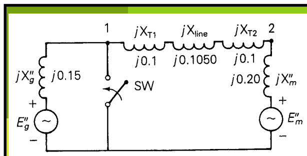
## PROBLEM 7.11




(a) Three-phase short circuit

Generator:

$E''_g = V_t + jI_L X''_d$ — voltage behind the subtransient reactance, or subtransient internal voltage

$E'_g = V_t + jI_L X'_d$ — voltage behind the transient reactance, or transient internal voltage

Motor:

inertia of rotor and field energized.
⇒ acts as a generator

$E''_m = V_t - jI_L X''_d$

$E'_m = V_t - jI_L X'_d$

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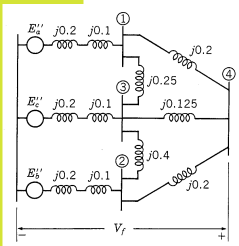
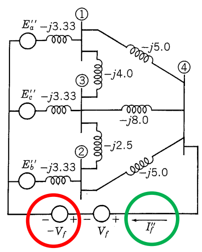
# Bus Impedance Matrix

$\mathbf{Y}_{\text{bus}}\mathbf{V} = \mathbf{I}$

$\mathbf{V} = \mathbf{Z}_{\text{bus}}\mathbf{I}, \quad \mathbf{Z}_{\text{bus}} = \mathbf{Y}_{bus}^{-1}$





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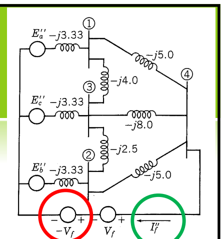

$\mathbf{I} = \mathbf{Y}_{\text{bus}}\mathbf{V}$

Admittances rather than impedances have been marked in per unit on this diagram. If $E_a''$, $E_b''$, $E_c''$, and $V_f$ are short-circuited, the voltages and currents are those due only to $-V_f$. Then the only current entering a node from a source is that from $-V_f$ and is $-I_f''$ *into* node 4 ($I_f''$ *from* node 4) since there is no current in this branch until the insertion of $-V_f$. The node equations in matrix form for the network with $-V_f$ the only source are

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ -I_f'' \end{bmatrix} = j\begin{bmatrix} -12.33 & 0.0 & 4.0 & 5.0 \\ 0.0 & -10.83 & 2.5 & 5.0 \\ 4.0 & 2.5 & -17.83 & 8.0 \\ 5.0 & 5.0 & 8.0 & -18.0 \end{bmatrix}\begin{bmatrix} V_1^{\Delta} \\ V_2^{\Delta} \\ V_3^{\Delta} \\ -V_f \end{bmatrix} \qquad (10.11)$$

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$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ -I_f'' \end{bmatrix} = j\begin{bmatrix} -12.33 & 0.0 & 4.0 & 5.0 \\ 0.0 & -10.83 & 2.5 & 5.0 \\ 4.0 & 2.5 & -17.83 & 8.0 \\ 5.0 & 5.0 & 8.0 & -18.0 \end{bmatrix}\begin{bmatrix} V_1^{\Delta} \\ V_2^{\Delta} \\ V_3^{\Delta} \\ -V_f \end{bmatrix} \qquad (10.11)$$

By inverting the bus admittance matrix of the network of Fig. 10.12 we obtain the bus impedance matrix. The bus voltages due to $-V_f$ are given by

$$\begin{bmatrix} V_1^{\Delta} \\ V_2^{\Delta} \\ V_3^{\Delta} \\ -V_f \end{bmatrix} = \mathbf{Z}_{\text{bus}}\begin{bmatrix} 0 \\ 0 \\ 0 \\ -I_f'' \end{bmatrix}$$

$$\mathbf{Z}_{\text{bus}} = \mathbf{Y}_{bus}^{-1}$$

$$\mathbf{Z}_{\text{bus}} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix}$$

and so

$$I_f'' = \frac{V_f}{Z_{44}} \qquad (10.13)$$

and

$$V_1^{\Delta} = -I_f'' Z_{14} = -\frac{Z_{14}}{Z_{44}} V_f$$

$$V_2^{\Delta} = -\frac{Z_{24}}{Z_{44}} V_f \qquad V_3^{\Delta} = -\frac{Z_{34}}{Z_{44}} V_f \qquad (10.14)$$

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$$V_1 = V_f + V_1^\Delta = V_f - I_f'' Z_{14}$$
$$V_2 = V_f + V_2^\Delta = V_f - I_f'' Z_{24}$$
$$V_3 = V_f + V_3^\Delta = V_f - I_f'' Z_{34} \qquad (10.15)$$
$$V_4 = V_f - V_f = 0$$

These voltages exist when subtransient current flows and $\mathbf{Z}_{\text{bus}}$ has been formed for a network having subtransient values for generator reactances.

In general terms for a fault on bus $k$, and neglecting prefault currents,

$$I_f = \frac{V_f}{Z_{kk}} \qquad (10.16)$$

and the postfault voltage at bus $n$ is

$$V_n = V_f - \frac{Z_{nk}}{Z_{kk}} V_f \qquad (10.17)$$

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Using the numerical values of Eq. (10.11), we invert the square matrix $\mathbf{Y}_{\text{bus}}$ of that equation and find

$\mathbf{Z}_{\text{bus}} = \mathbf{Y}_{bus}^{-1}$

$$\mathbf{Z}_{\text{bus}} = j\begin{bmatrix} 0.1488 & 0.0651 & 0.0864 & 0.0978 \\ 0.0651 & 0.1554 & 0.0799 & 0.0967 \\ 0.0864 & 0.0798 & 0.1341 & 0.1058 \\ 0.0978 & 0.0967 & 0.1058 & 0.1566 \end{bmatrix} \qquad (10.18)$$

Usually $V_f$ is assumed to be $1.0\underline{/0^\circ}$ per unit, and with this assumption for our faulted network

$$I_f'' = \frac{1}{j0.1566} = -j6.386 \text{ per unit}$$

$$V_1 = 1 - \frac{j0.0978}{j0.1566} = 0.3755 \text{ per unit}$$

$$V_2 = 1 - \frac{j0.0967}{j0.1566} = 0.3825 \text{ per unit}$$

$$V_3 = 1 - \frac{j0.1058}{j0.1566} = 0.3244 \text{ per unit}$$

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Currents in any part of the network can be found from the voltages and impedances. For instance, the fault current in the branch connecting nodes 1 and 3 flowing toward node 3 is

$$I_{13}'' = \frac{V_1 - V_3}{j0.25} = \frac{0.3755 - 0.3244}{j0.25}$$
$$= -j0.2044 \text{ per unit}$$

From the generator connected to node 1 the current is

$$I_a'' = \frac{E_a'' - V_1}{j0.3} = \frac{1 - 0.3755}{j0.3}$$
$$= -j2.0817 \text{ per unit}$$

Other currents can be found in a similar manner, and voltages and currents with the fault on any other bus are calculated just as easily from the impedance matrix.

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## RAKE EQUIVALENT

† This equivalent network is drawn in the manner adopted in J. R. Neuenswander, *Modern Power Systems*, Intext Educational Publishers, New York, 1971, which refers to the bus impedance matrix equivalent network as the *rake equivalent*.

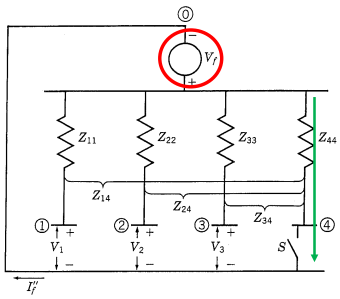


**Figure 10.13** Bus impedance matrix equivalent network with four independent nodes. Closing switch $S$ simulates a fault on node 4. Only the transfer admittances for node 4 are shown.

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## EXAMPLE – 5-BUS

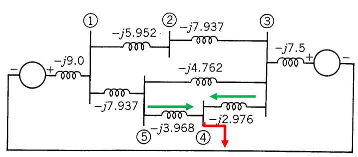


**Figure 10.14** Admittance diagram for Example 10.4.

The network with admittances marked in per unit is shown in Fig. 10.14 from which the node admittance matrix is

$$\mathbf{Y}_{\text{bus}} = j\begin{bmatrix} -22.889 & 5.952 & 0.0 & 0.0 & 7.937 \\ 5.952 & -13.889 & 7.937 & 0.0 & 0.0 \\ 0.0 & 7.937 & -23.175 & 2.976 & 4.762 \\ 0.0 & 0.0 & 2.976 & -6.944 & 3.968 \\ 7.937 & 0.0 & 4.762 & 3.968 & -16.667 \end{bmatrix}$$

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The network with admittances marked in per unit is shown in Fig. 10.14 from which the node admittance matrix is

$$\mathbf{Y}_{\text{bus}} = j\begin{bmatrix} -22.889 & 5.952 & 0.0 & 0.0 & 7.937 \\ 5.952 & -13.889 & 7.937 & 0.0 & 0.0 \\ 0.0 & 7.937 & -23.175 & 2.976 & 4.762 \\ 0.0 & 0.0 & 2.976 & -6.944 & 3.968 \\ 7.937 & 0.0 & 4.762 & 3.968 & -16.667 \end{bmatrix}$$

This 5 × 5 bus is inverted on a digital computer to yield the short-circuit matrix

$$\mathbf{Z}_{\text{bus}} = j\begin{bmatrix} 0.0793 & 0.0558 & 0.0382 & 0.0511 & 0.0608 \\ 0.0558 & 0.1338 & 0.0664 & 0.0630 & 0.0605 \\ 0.0382 & 0.0664 & 0.0875 & 0.0720 & 0.0603 \\ 0.0511 & 0.0630 & 0.0720 & 0.2321 & 0.1002 \\ 0.0608 & 0.0605 & 0.0603 & 0.1002 & 0.1301 \end{bmatrix}$$

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$$\mathbf{Z}_{\text{bus}} = j\begin{bmatrix} 0.0793 & 0.0558 & 0.0382 & 0.0511 & 0.0608 \\ 0.0558 & 0.1338 & 0.0664 & 0.0630 & 0.0605 \\ 0.0382 & 0.0664 & 0.0875 & 0.0720 & 0.0603 \\ 0.0511 & 0.0630 & 0.0720 & 0.2321 & 0.1002 \\ 0.0608 & 0.0605 & 0.0603 & 0.1002 & 0.1301 \end{bmatrix}$$

Visualizing a network like that of Fig. 10.13 will help in finding the desired currents and voltages.

The subtransient current in a three-phase fault on bus 4 is

$$I'' = \frac{1.0}{j0.2321} = -j4.308 \text{ per unit}$$

At buses 3 and 5 the voltages are

$$V_3 = 1.0 - (-j4.308)(j0.0720) = 0.6898 \text{ per unit}$$

$$V_5 = 1.0 - (-j4.308)(j0.1002) = 0.5683 \text{ per unit}$$

Currents to the fault are

From bus 3: $0.6898(-j2.976) = -j2.053$

From bus 5: $0.5683(-j3.968) = \underline{-j2.255}$

$-j4.308$ per unit

From the same short-circuit matrix we can find similar information for faults on any of the other buses.

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